Variable density groundwater flow: Are equivalent freshwater heads necessary or misleading?

James C. Ellis and K. Udo Weyer
WDA Consultants Inc., Calgary, Alberta, Canada

ABSTRACT
Variable density flow of subsurface fluids, such as fresh water, brackish water, and brines, may occur in areas with salt layers, salty tailings, up-coning effects of saline water, contamination, as well as in deep groundwater flow systems and in hydrocarbon reservoirs. There are a number of computer programs available (SUTRA and others) all purporting to be able to calculate adequate flow patterns for freshwater and saltwater. These programs make use of velocity potentials [energy/unit volume] to determine gradients for subsurface flow. The use of velocity potentials requires three basic assumptions: (1) the energy within the gravitational field relates to unit volumes, (2) underground fluids are incompressible, and (3) equivalent fresh water heads stand for the actual energy conditions in a flow field. Equivalent freshwater heads do not, however, correctly represent the energy conditions in flow fields in the subsurface and all underground fluids are compressible.
The above assumptions are not necessary when flow calculations are based on force potentials [energy/unit mass]. As the mass is measured in kilograms and a mass of 1 kg is independent of pressure, density, and temperature of the fluid the actual heads measured in piezometers containing fluids of any density, compression, or temperature are the correct head values and can directly be used in flow calculations by programs based on force potentials. Thereby the use of equivalent fresh water heads is unnecessary and misleading. When using force potentials, buoyancy forces can be directed in any direction in space and are integrated in the resultant calculation for the head and density-dependent pressure potential forces driving variable density subsurface flow, together with gravitational forces under heterogeneous, hydrodynamic conditions.
Clear and simple diagrams will demonstrate the differences between the two approaches and the advantages of using mathematically- and physically-correct force potentials over only mathematically-correct velocity potentials.